



Preliminary Study of PIC using ICOOL

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This study is aimed at understanding the PIC scheme as seen by the absorber. How the lattice delivers the beams to the absorber is not studied in detail. The betas refer to those of the beam ($\beta_{x,y} = \sigma_{x,y}/\sigma_{x',y'}$ at a focus), and not of the lattice. ICOOL is used to observe cooling and heating in a few early absorbers, and these results are scaled to obtain cooling and heating for the complete system. Parameters are initially taken from a revised copy of Derbenev and Johnson's paper on "Parametric-resonance Ionization Cooling and Reverse Emittance Exchange for Muon Colliders." Ideal emittance exchange is added to correct the observed longitudinal emittance growth. An optimization of the angular spreads and corresponding betas is undertaken.

Parameters from the PIC/REMEX revised paper

		Pic 1	PIC 2
Cell lengths	cm	19	19
Momentum	MeV/c	100	100
Muons/bunch		10^{11}	10^{11}
Absorber thick	mm	6.4	1.6
Absorber Mat		Be	Be
Trans RMS emit	mm mrad	600	30
Sigma(theta)	Mrad	200	200
Sigma(r)	mm	3	0.15
Beta	mm	15	0.75
Equiv B	T	44.4	888
ϵ_o	mm mrad	118	6.0
RMS dp/p	%	3	3
Sigma(z)	cm	0.5	0.5
Long RMS emittance	cm	0.015	0.015

$$\beta_{\perp} = \frac{\sigma_{x,y}}{\sigma_{\theta x, \theta y}} = 15 \text{ mm}$$

$$B_{\text{effective}} = \frac{2 p}{c_{\text{vel}} \beta_{\perp}} \approx 44.4 \text{ T}$$

$$\epsilon_o = \frac{\beta_{\perp}}{\beta_v} C_{\text{Be}} \frac{dEdx(\text{min})}{dEdx(p)} \approx 118 \quad 10^{-6} \text{ m}$$

The blue numbers differ from the original paper

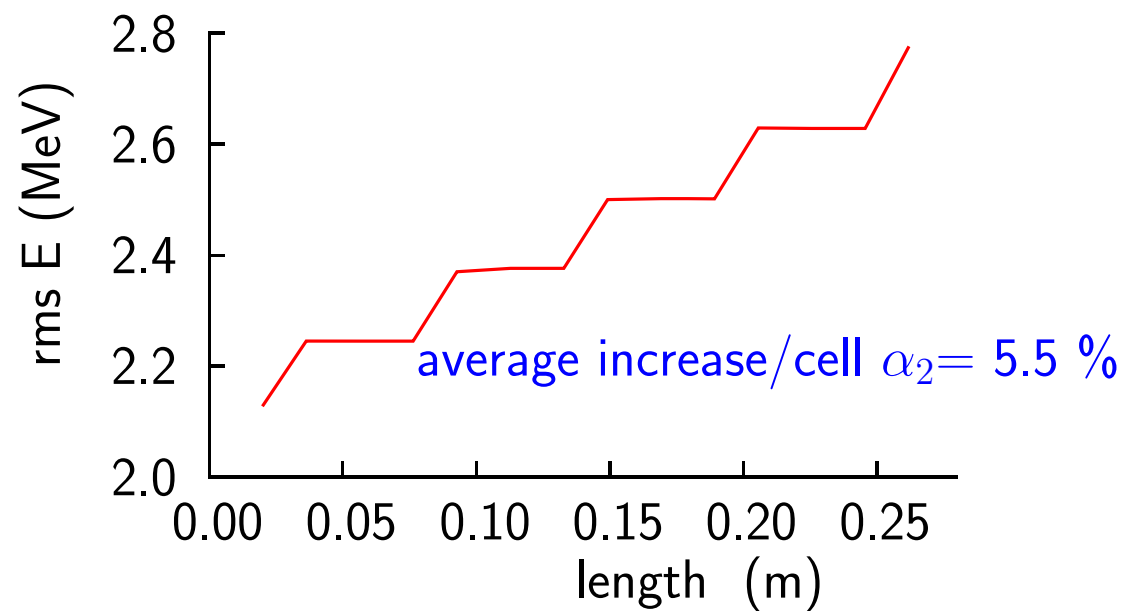
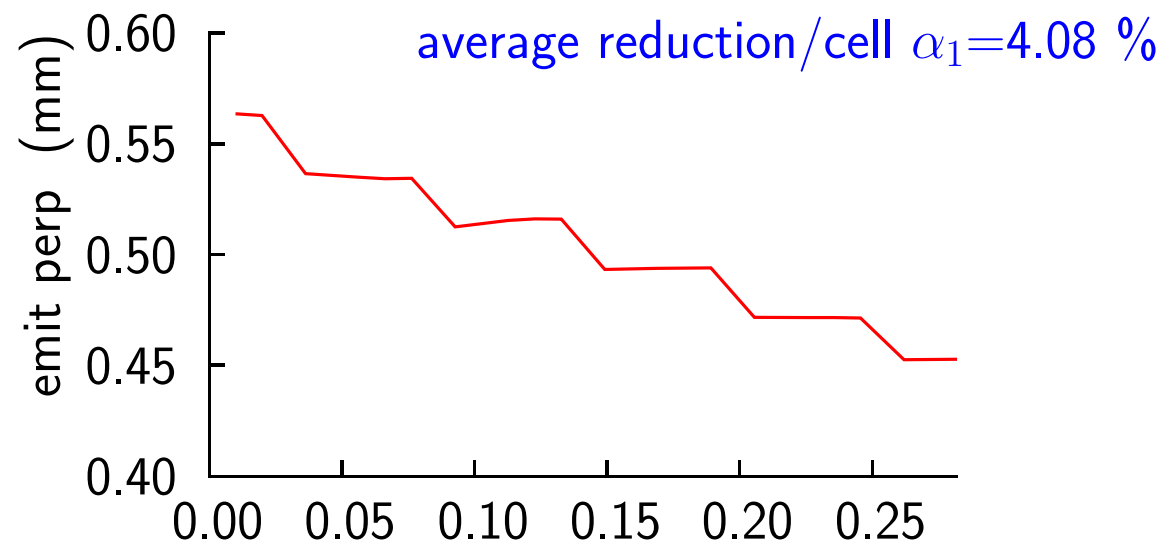
The red numbers are calculated on right

- The "Equivalent Betas" are those to give the required beta continuously
- The equilibrium emittance " ϵ_o " was checked using ICOOL

Method of Study

- Use axial magnetic field of 44 T to mimic $\beta = 15$ mm foci
- Insert 5 Be absorbers with 6.4 mm thickness
alternating with 5 "magic" acceleration to restore energy lost
- Run two ICOOL decks with 1000 particles each
 1. Run ICOOL to get transverse cooling
Input Gaussian data with $\sigma_\theta = 0.2$ radians, $\sigma_r = 3$ mm, and $dp/p = 3\%$
 2. Run ICOOL to get longitudinal heating
Input Gaussian data with $\sigma_\theta = 0$, $\sigma_r = 3$, and $dp/p = 3\%$
This avoids the non-heating effect of different path length at different angles
- Plot transverse emittance and RMS sigma E
dt is not changed by an interaction, so $\sigma_E \propto \epsilon_{||}$ (long emittance)
- Extract average fractional decreases in transverse emittance α_1
and fractional increases in longitudinal emittance α_2
- Use these data to predict cooling down to 30 pi mm

Output from ICOOL runs



Details of calculations of cooling down to 30 pi mm

The absorber thicknesses

$$t = 6.4 \text{ (mm)}$$

or

$$t = \frac{1.6}{0.75} \times \epsilon_{\perp}$$

which ever is smaller

The emittances for each succeeding cell are calculated from:

$$\begin{aligned}\epsilon_{\parallel}(n+1) &= \epsilon_{\parallel}(n) \times \frac{t}{6.4mm} \times \alpha_2 \\ \epsilon_{\perp}(n+1) &= \epsilon_{\perp}(n) \times \frac{t}{6.4mm} \times \alpha_1 \times k\end{aligned}$$

Where k is a correction for effects of the rise in beta away from the focus

Calculation of k (correction for beta's rise away from focus)

$$\beta = \beta_o \left(1 + \frac{z^2}{\beta_o^2} \right)$$

If L is the half length of the absorber then $\eta = L/\beta_o$

$$f = \frac{\langle \beta \rangle}{\beta_o} = \left(1 + \frac{\eta^2}{3} \right)$$

From the above table [$\epsilon_o/\epsilon=170/600$ (initial) = $8.5/30$ (final) = .283]

$$\frac{d\epsilon_{\perp}}{\epsilon_{\perp}} = \frac{dp}{p} \left(1 - \frac{\epsilon_o}{\epsilon_{\perp}} \right) = \frac{dp}{p} \left(1 - \frac{170}{600} f \right)$$

so

$$k = \left(\frac{1 - .283 f}{1 - .283 f_o} \right)$$

Initially, $L=6.4/2=3.2$ (mm) and $\beta_o=15$ mm, so $\eta_o = 0.21$ and

$$f_o = \frac{\langle \beta \rangle}{\beta_o} = 1.015 \quad k_o \approx 1.00$$

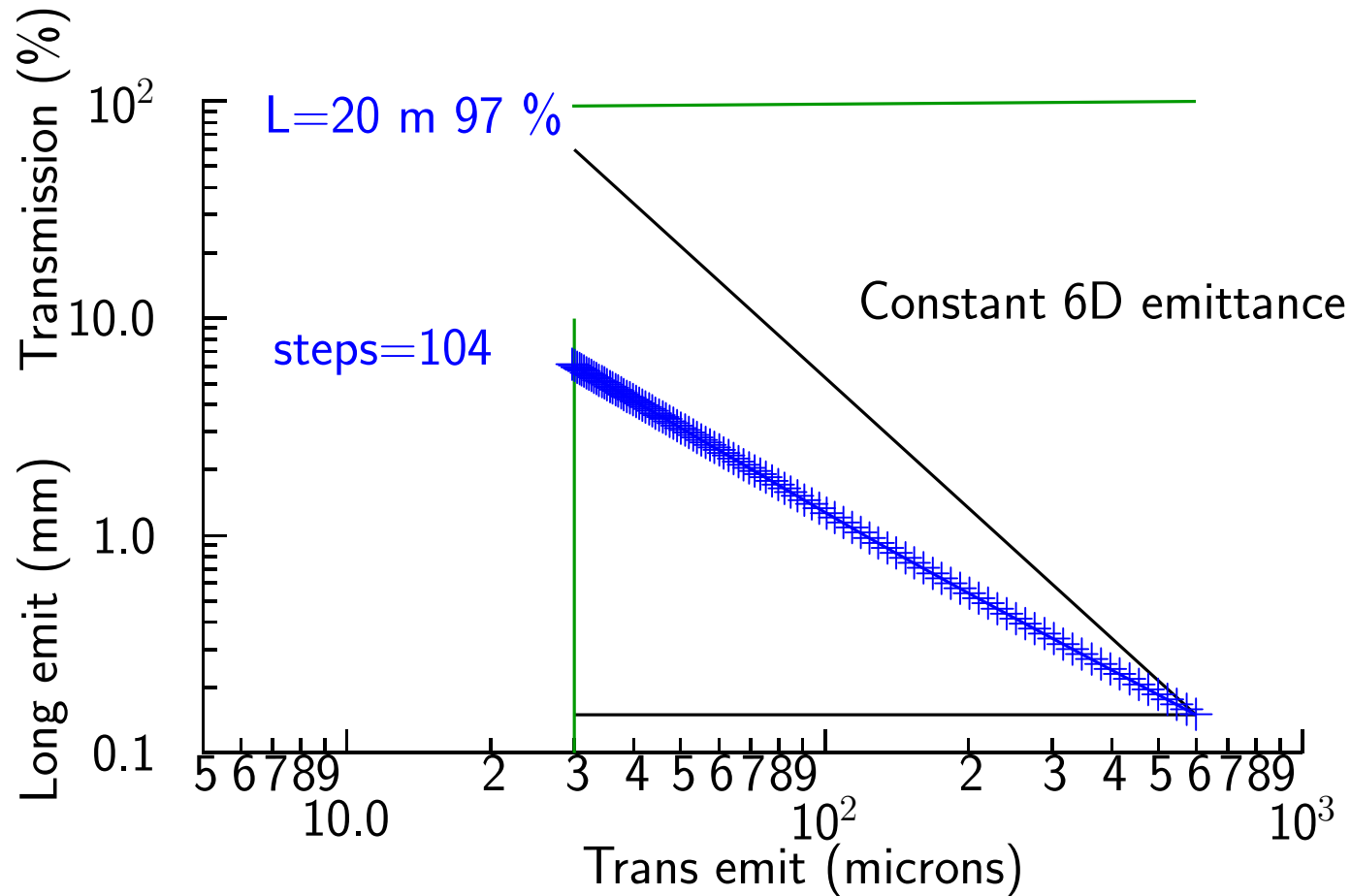
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but at the end $L=1.6/2=0.8$ mm and $\beta_n = 0.75$ mm, so $\eta_n = 1.07$ and

$$k_n = \frac{\langle \beta \rangle}{\beta_o} = 1.38 \quad k_n \approx 0.85$$

Longitudinal vs. transverse emittances

- With cell lengths = 19 cm
As specified in paper
- No emittance exchange

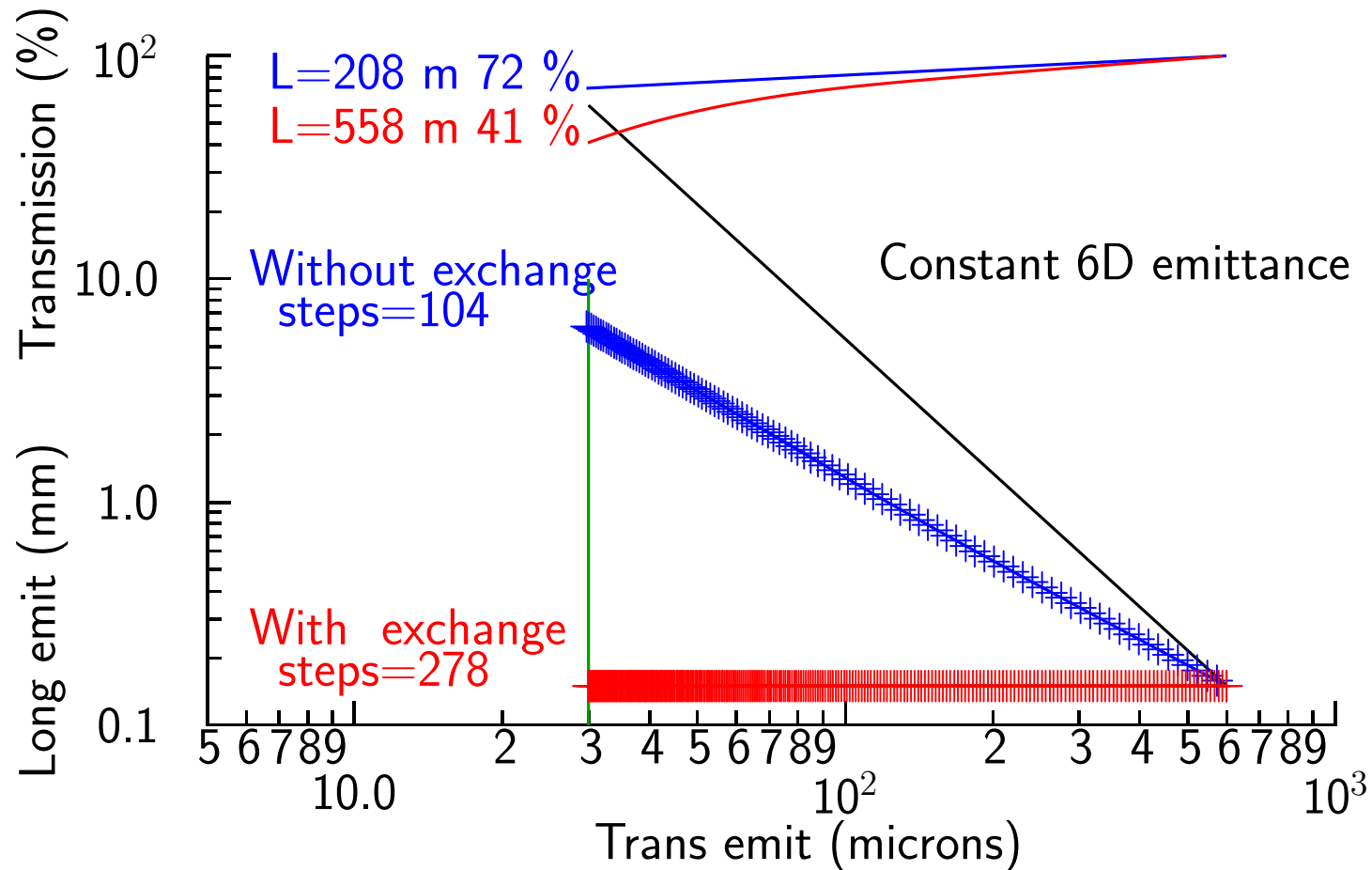


Comments

- Straggling and dE/dx slope effects cause increase in long emittance
We can fix this with emittance exchange: assumed ideal
- We note that the 19 cm cell length assumed an un-chromatically corrected lattice
We will instead use 2 m, as now being studied

With Emittance Exchange and cell=2 m

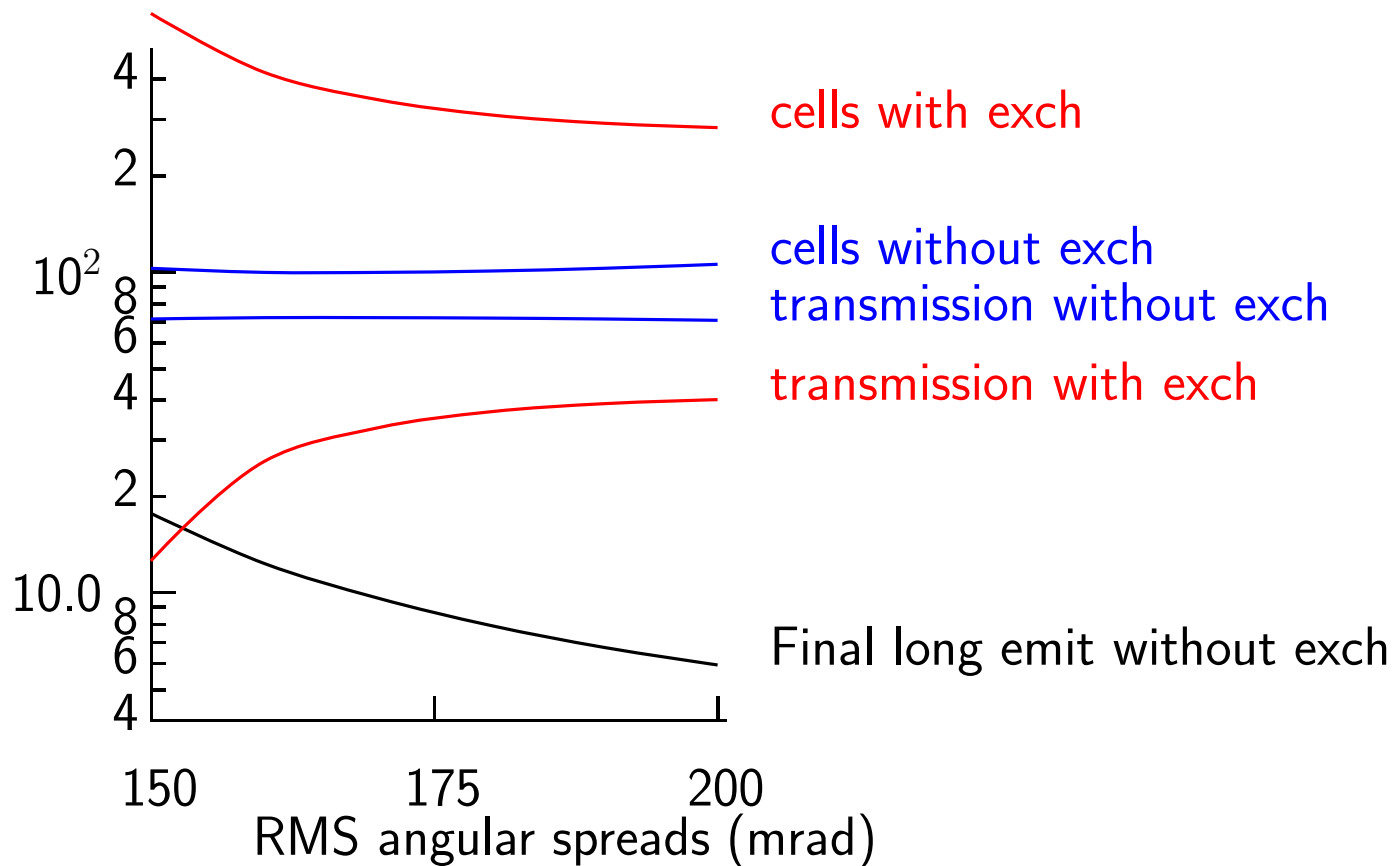
- With cell lengths = 2 m
- With ideal emittance exchange to hold Long emittance constant



- The channel now has a length of 558 m and 41% transmission from decay

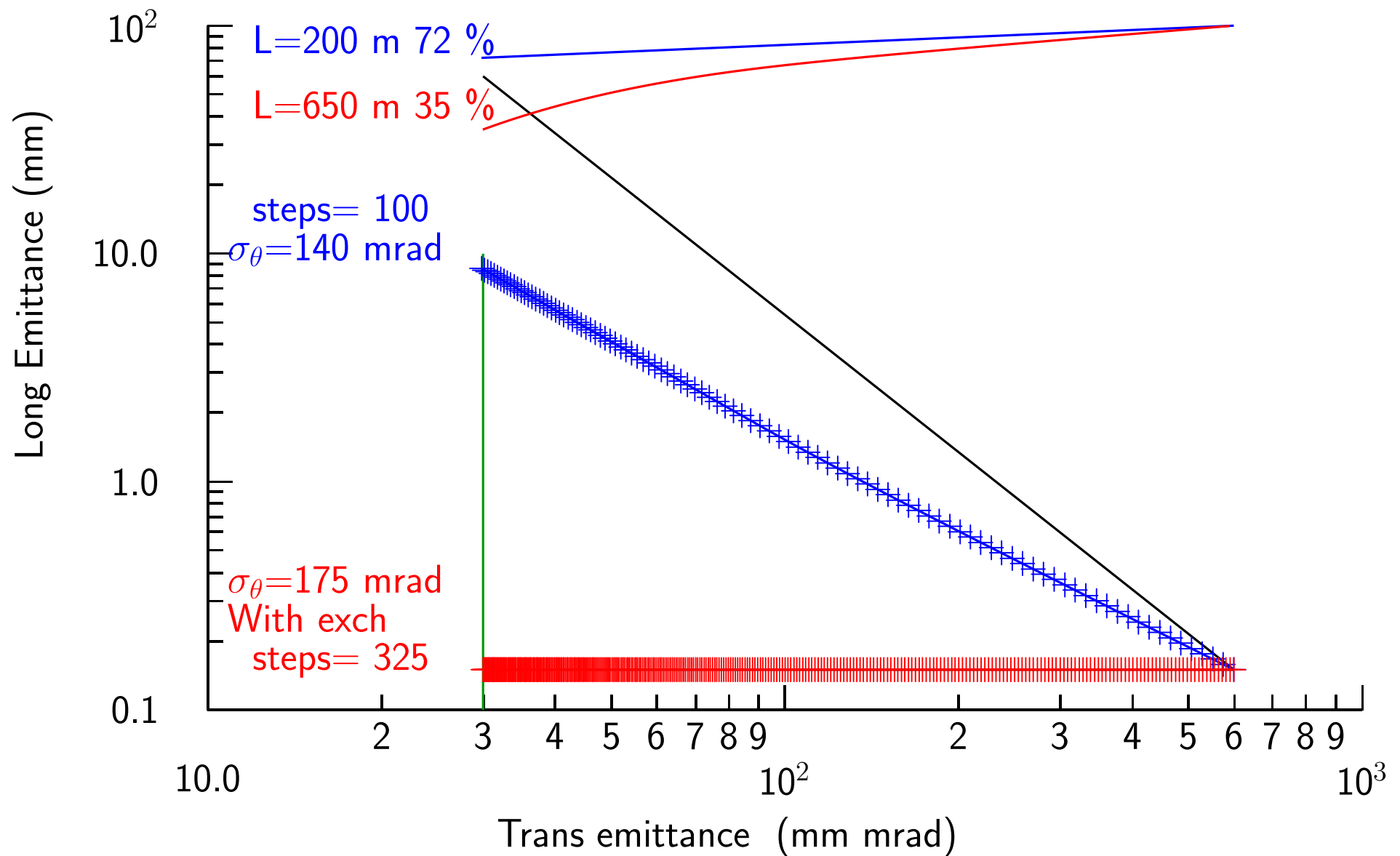
Dependence on angle spread

- The ratio of emittance to equilibrium emittance is rather large (5.08)
- Corresponding to a very large angular spread (200 mrad)
- And very small betas (15-.75 mm)
- If we reduce these spreads and increase the betas then
 - The transverse cooling per energy lost will be worse
 - But the absorbers can be longer giving more cooling per cell
- So it is interesting to study dependencies



- Without exchange there is a minimum number of cells and loss for spread of ≈ 160 mrad but in this case the final longitudinal emittance is greater (12 vs 7 (pi mm))
- With emittance exchange number of cells rises and transition falls but with 175 mrad the changes are modest (350 vs 278, 35% vs 41%)
- Use of 175 mrad spread (420 mrad at 3 sigma) will make the lattice a little easier

Performance of 175 mrad case



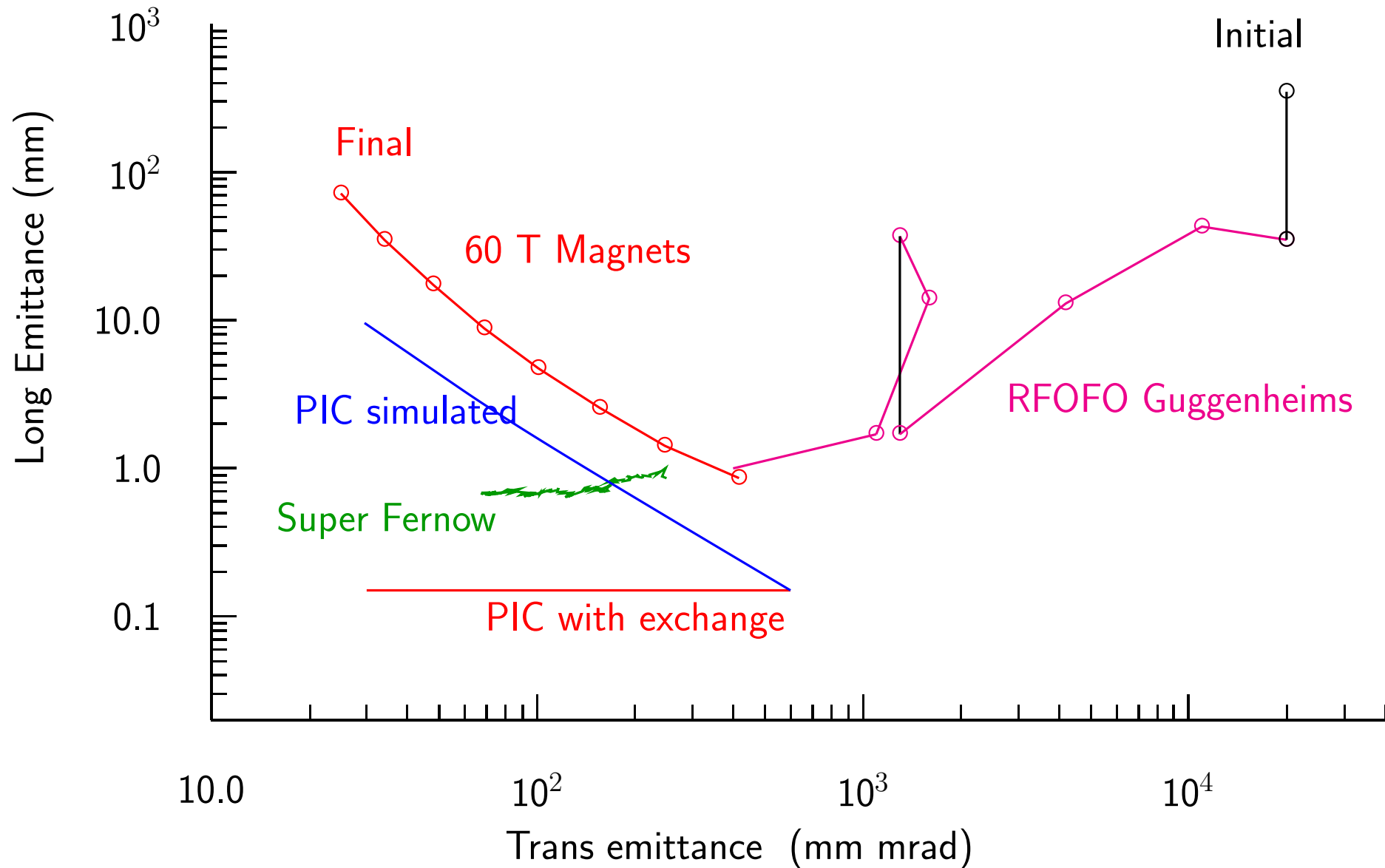
Parameters from this study

Parameters of simulated "Super Fernow" lattice shown for comparison

		DJ ₁	DJ ₂	RBP ₁	Without exch ₂	With exch ₂	Super Fernow ₂
Cell lengths	cm	19	19	200	200	200	45
Momentum	MeV/c	100	100	100	100	100	200
Muons/bunch		10 ¹¹	10 ¹¹	10 ¹¹	0.72 10 ¹¹	0.35 10 ¹¹	20 10 ¹¹
Absorber thick	mm	6.4	1.6	6.4	2.1	2.1	10
Wedge angle	Deg	0	0	0	0	?	90
Absorber Mat		Be	Be	Be	Be	Be	LiH
Trans RMS emit	mm mrad	600	30	600	30	30	68
Sigma(theta)	Mrad	200	200	175	175	175	60
Sigma(r)	mm	3	0.15	3.4	0.17	0.17	0.6
Beta	mm	15	0.75	19.5	1.0	1.0	10
Equiv B	T	44	880	34	670	670	66
ε _o	mm mrad	118	6	153	8.0	8.0	60
RMS dp/p	%	3	3	3	3	3	3
Sigma(z)	cm	0.5	0.5	0.5	33	.5	1.1
Long RMS emittance	cm	0.015	0.015	0.15	10	0.15	0.7
Dispersion	cm	0	0	0	0	?	1.0

Red numbers show differences from paper

Plot together with some other simulations



The 50 T magnets with 5 times rf freq would be similar to PIC, for good reason.
Both use lower energies to lower ϵ_{\perp} at the expense of ϵ_{\parallel}

Conclusions

- Transmission can be slightly improved by lowering the specified RMS angular spreads (from 200 to 175 mrad) and correspondingly increasing the betas.
- Without emittance exchange the PIC suffers strong longitudinal emittance growth (from 0.15 to 10 π mm) leading to performance not much different from cooling at low energies with hydrogen in 50 T solenoids.
- The number of cells required is 100, giving a length (for 2 m cells) of 200 m and transmission from decay of 72%
- Transverse cooling in the final cell is 1.8 %
- If ideal emittance exchange is introduced to maintain a constant longitudinal emittance, then the required number of cells rises to 325 for an estimated length (2 m cells) of 650 m, giving transmission from decay of only 35%
- Transverse cooling in the final cell, is now only 0.46 % making it very sensitive to any emittance growth in the lattice

Next Steps

- Study dependence on PIC momentum
- Add dispersion and wedge absorbers to the above PIC scheme to simulate the emittance exchange
- Study whether an angular acceptance of 525 mrad (for 3 sigma) and betas of 1mm are possible. The best achieved in the BNL "Super Fernow" lattice was an acceptance of 180 mrad (at 3 sigma) and beta of 1 cm, and this lattice contained no non-linear elements like sextupoles to correct chromatic effects.
- Pick parameters for a REMEX simulation
 - I could try keeping the beta constant and equal to its value at the end of the above PIC with exchange
 - It would clearly work better if I continued to taper the beta, keeping the angular spread constant, but this requires even more exotic lattice specifications
 - Advice from Derbenev and Johnson, as to what they assumed, would be appreciated
- I could then simulate REMEX using ICOOL